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Closed Strings Interacting with Noncommutative D-branes

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ABSTRACT

Closed string dynamics in the presence of noncommutative Dp-branes is investigated. In particular, we compute bulk closed string two-point scattering amplitudes; the bulk space-time geometries encoded in the amplitudes are shown to be consistent with the recently proposed background space-time geometries dual to noncommutative Yang-Mills theories. Three-point closed string absorption/emission amplitudes are obtained to show some features of noncommutative Dp-branes, such as modified pole structures and exponential phase factors linearly proportional to the external closed string momentum.

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1 Introduction

In the framework of open strings, it is now possible to systematically study the physics on noncommutative space-time [1]-[7]. In particular, a natural vantage point for the investigation of $(p+1)$-dimensional gauge theories on a noncommutative space is to consider the world-volume theory of Dp-branes when the constant background NS-NS two-form gauge fields parallel to the branes are turned on. One can then study open string dynamics stuck to the branes and find that, in an appropriate decoupling limit, the world-volume theory corresponds to noncommutative Yang-Mills theory [1]-[7].

The main theme of this paper is to understand closed string dynamics in the presence of noncommutative Dp-branes. On top of an obvious observation that any theory of open strings should include closed strings, there are other reasons to study closed string dynamics in this context. By now, there are considerable body of evidence toward the validity of the duality between the conformal field theory on a $(p+1)$-dimensional space and the string theory/supergravity on a $(p+2)$-dimensional Anti-de Sitter space [8]. One natural question following the understanding of noncommutative Yang-Mills theory is to find an appropriate dual background space-time geometry. Recently, such dual background space-time geometries were proposed by Hashimoto and Itzhaki [9], and Maldacena and Russo [10], following the chain of T-duality arguments [1]. These background space-time geometries can be directly probed by closed strings moving on it. Thus, the study of the closed string two-point scattering amplitudes shown in Fig. 1 may provide us with a direct string theoretic justification for the proposed background geometries. The second issue is to understand noncommutative D-brane black holes. It has been noted that the Hawking radiation from D-brane black holes and its time-reversal process, the matter absorption into such black holes, can be understood via the microscopic D-brane description by considering three-point amplitudes of the type shown in Fig. 2 [11]. One then wonders what will be the effect of the world-volume noncommutativity on the emission/absorption processes from D-branes. Finally, it is suggested that the noncommutative gauge theories play a crucial role in the development of string field theories [7, 12]. Understanding subleading effects in $1/N$ is important in the construction of string field theories, and the inclusion of the closed strings is an essential step for that purpose; one needs to evaluate correlation functions on general Riemann surfaces with boundaries along with marked points and handles. It will be nice to have a simple calculational prescription to take into account of the effect of the background NS-NS two-form gauge fields in such a context[1]. In this paper, we address the first two issues and, in that process, make a modest progress toward the third issue.

In Section 2, after setting up our notations based on Refs. [13, 14], we explain how the noncommutativity effects are implemented by simple global rotations of the two string coordinates parameterizing a two-cycle along which the NS-NS two-form gauge field $B_{\mu\nu}$ has a non-vanishing component whose strength determines the global rotation angle. Starting from the usual Neumann boundary conditions where we identify the holomorphic sector and the anti-holomorphic sector at the world-sheet boundary, the operator products for the general value of $B_{\mu\nu}$ can thus be simply determined. The senses of the rotation for the holomorphic sector and the anti-holomorphic sector are opposite to each other; therefore, the $\pi/2$ rotation (large $B_{\mu\nu}$ limit) flips the Neumann boundary condition into the Dirichlet boundary condition, turning Dp-branes into D$(p-2)$-branes. For generic values of $B_{\mu\nu}$, both Dp-branes and D$(p-2)$-branes are present [15]. The correlation functions in the presence of noncommutative D-branes can be computed by inserting these rotation matrices at appropriate steps of the calculations.

In section 3, we directly compute the tree-level closed string two-point scattering amplitudes, shown in Fig. 1 in the presence of noncommutative Dp-branes. In the s-channel factorization limit, intermediate open string states are identified and are shown to be consistent with the picture of Seiberg and Witten [7]. On-shell, this amplitude does not contain (due to the momentum conservation parallel to the branes) the exponential phase factor used to define the $*$-product [7, 16].

In section 4, the same scattering problems are also investigated on the background geometries of Refs. [9, 10] via supergravity analysis at long distance (Fig. 2). The results turn out to be identical to those obtained in Section 3 when we approach the leading t-channel pole for the string amplitudes, justifying the background supergravity space-time geometries of Refs. [9, 10].

In section 5, we compute the three-point amplitudes shown in Fig. 3 to study the absorption/emission from noncommutative Dp-branes. The key difference between the commutative and the noncommutative cases is the existence of an exponential phase factor for each factorization channel, which is used to define the $*$-product in the noncommutative case [7]. By the momentum conservation parallel to the branes, the exponential phase depends linearly on the external closed string momentum parallel to the branes; it vanishes for the external closed string momentum with vanishing parallel components (see also [17]). Furthermore, when compared to

[1] In the case of purely open string diagrams, a simple prescription in this regard is available in Refs. [5, 7].

the commutative case, the pole and zero structures of the amplitudes are distinctively changed.

In Section 6, we discuss the implications and possible generalizations of our calculations presented in this paper.

2 Preliminaries

Our computation of correlation functions will be based on the modern covariant formulation defined via a conformal field theory on the open string world-sheet. Our primary interest is to understand how the existence of the background NS-NS two-form gauge field affects the correlation functions involving closed strings. In the context of pure open strings, this issue was investigated in [5, 7]. A useful starting point for this purpose is to consider the open string action

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma(g_{\mu\nu}\partial^a X^\mu \partial_a X^\nu + iB_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu) + \text{fermionic part}, \tag{2.1}$$

where X^μ, $g_{\mu\nu}$ and $B_{\mu\nu}$ represent the string coordinates, the constant background metric and the constant background NS-NS two-form field, respectively. The background B field has non-zero components only along the directions parallel to the Dp-branes. The components perpendicular to the branes can be gauged away.

Since the B field vanishes along the directions perpendicular to the branes, we can simply impose the usual Dirichlet boundary conditions for the perpendicular directions. The parallel directions, however, even if the bulk equations of motion are not affected by the constant B field, the Neumann boundary condition changes to

$$g_{\mu\nu}\partial_n X^\nu + iB_{\mu\nu}\partial_t X^\nu = 0 \tag{2.2}$$

on the open string end points. Here ∂_n and ∂_t denote the normal derivative and the tangential derivative to the boundary of the string world sheet, respectively.

In this paper, we will be interested in disk diagrams only. For the computational convenience, we map the disk to the upper half plane, putting boundary at the real-axis on the complex plane. Also for simplicity, we set $g_{\mu\nu} = \eta_{\mu\nu}$ [2]. It is straightforward to relax this restriction.

A useful trick to handle the mode expansion under the boundary condition Eq. (2.2) is the following. Let $r = 2k$ be the rank of the background B field. Using the $SO(1,p)$ symmetry

[2] When writing down products of tensor objects, especially from Section 3, the dot product with respect to $g^{\mu\nu}$ is implied, unless otherwise noted.

of the brane and redefining some coordinates, we can set $B_{\mu\nu} = 0$ for $0 \leq \mu \leq p-r$ or $0 \leq \nu \leq p-r$ and bring the remaining $r \times r$ matrix to the block diagonal form. If we denote the r coordinates by y_i ($1 \leq i \leq r$), the restriction of B field to (y_{2i-1}, y_{2i}) subspace takes the form

$$B_i = \begin{pmatrix} 0 & B_i \\ -B_i & 0 \end{pmatrix}. \tag{2.3}$$

Introduce a matrix R whose i-th 2×2 block is

$$R_i = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} , \tag{2.4}$$

where $\theta_i \equiv \tan^{-1} B_i$, and otherwise equal to the identity matrix [3]. In terms of the matrix R, the boundary condition (2.2) can be rewritten as

$$R^T \partial_z X - R \partial_{\bar{z}} X = 0, \tag{2.5}$$

where we consider X^μ as a column vector.

Let $X(z)$ be the operator defined in terms of the mode expansion [4]

$$\partial X^\mu(z) = -i \sum_{n=-\infty}^{\infty} \alpha_n^\mu z^{-n-1} . \tag{2.6}$$

When $B_{\mu\nu} = 0$, we define the anti-holomorphic counterpart $X(\bar{z})$ using the *same* oscillator modes α_n^μ in order to ensure the Neumann boundary condition on $X(z,\bar{z}) = X(z) + X(\bar{z})$. When $B_{\mu\nu} \neq 0$, we note that the field

$$X(z,\bar{z}) = RX(z) + R^T X(\bar{z}) \tag{2.7}$$

satisfies (2.5). Using the standard operator product

$$\langle X^\mu(z) X^\nu(w) \rangle = -\eta^{\mu\nu} \ln(z-w), \tag{2.8}$$

one can easily determine the operator products for the "rotated" $X, \bar{X}$ fields:

$$\begin{aligned}
\langle (RX)^\mu(z)(RX)^\nu(w) \rangle &= -\eta^{\mu\nu} \ln(z-w), && (2.9)\\
\langle (R^T\bar{X})^\mu(\bar{z})(R^T\bar{X})^\nu(\bar{w}) \rangle &= -\eta^{\mu\nu} \ln(\bar{z}-\bar{w}), && (2.10)\\
\langle (RX)^\mu(z)(R^T\bar{X})^\nu \rangle &= -(2G^{\mu\nu} - \eta^{\mu\nu} + 2\Theta^{\mu\nu}) \ln(z-\bar{w}), && (2.11)
\end{aligned}$$

[3] The authors of Ref. [18] considered string scattering amplitude in the presence of *electric* flux on D-branes. They introduced a boost matrix as a function of the electric field, whose magnetic counterpart is our rotation matrix. They also observed phase factors analogous to ours (5.13). In addition, after the initial submission of this paper, M.R. Garousi informed us that the large part of the calculations presented in Sections 2 and 3 were already reported in Ref. [19], albeit in a different language.

[4] From here on, we set $\alpha' = 2$.

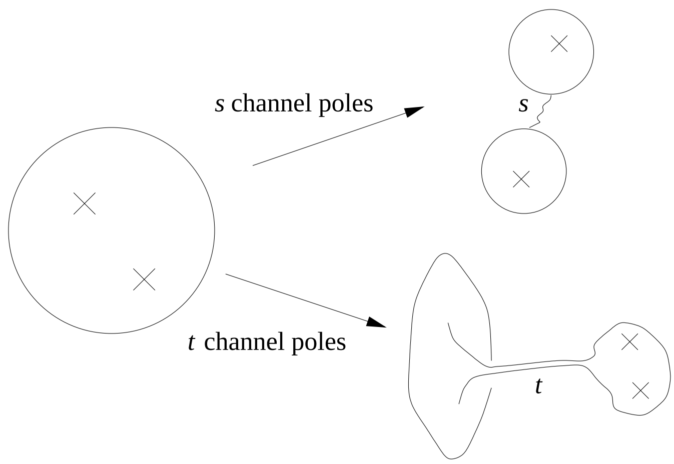

Figure 1: Two-point closed string diagram. On the right hand side, we show a t-channel and an s-channel factorization limit.

where we defined, following [7], $G^{\mu\nu}$ and $\Theta^{\mu\nu}$ as the symmetric and anti-symmetric part of $(\eta_{\mu\nu} + B_{\mu\nu})^{-1}$, respectively. Clearly, one can combine the above formulas to obtain the expression for the operator product of two open string vertex operators, originally derived in [14]

The holomorphic part and the anti-holomorphic part of the string coordinates are related to each other via

$$(RX(z)) = R^2 \left(R^T \bar{X}(\bar{z})\right) \tag{2.12}$$

at the boundary $z = \bar{z}$. Therefore, as $B_i \to \infty$, i.e., $\theta_i \to \pi/2$, the Neumann boundary condition at $\theta_i = 0$ flips to the Dirichlet boundary condition, $R_i^2 = -I$. In the intermediate case, the boundary conditions are mixed, $\tan\theta_i$ being the "measure of the relative proportion" of the Dirichlet boundary parts ("D$(p-2)$-branes") to the Neumann boundary parts ("Dp-branes").

3 Scatterings from noncommutative D-branes

We now perform the calculation of the bulk two-point closed string amplitudes shown in Fig. 1. After a brief review of the same calculation without the background B field [11], we turn on the constant background B field.

3.1 Review: Scatterings from commutative D-branes

The closed string scattering amplitude from a commutative Dp-brane is given by

$$A = \int d^2z_1 d^2z_2 \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2)\rangle, \tag{3.1}$$

corresponding to the string diagram Fig. 1. The appropriate vertex operators are

$$
\begin{aligned}
V_1(z_1,\bar{z}_1) &= (\varepsilon_1 D)_{\mu\nu} V_{-1}^{\mu}(p_1,z_1) V_{-1}^{\nu}(Dp_1,\bar{z}_1)\ , && (3.2)\\
V_2(z_2,\bar{z}_2) &= (\varepsilon_2 D)_{\mu\nu} V_{0}^{\mu}(p_2,z_2) V_{0}^{\nu}(Dp_2,\bar{z}_2)\ , && (3.3)\\
V_{-1}^{\mu}(p_1,z_1) &= e^{-\phi(z_1)}\psi^{\mu}(z_1) e^{ip_1X(z_1)}\ , && (3.4)\\
V_{0}^{\mu}(p_2,z_2) &= \{\partial X^{\mu}(z_2) + ip_2\cdot\psi(z_2)\psi^{\mu}(z_2)\} e^{ip_2X(z_2)}\ , && (3.5)
\end{aligned}
$$

where the matrix D is defined as

$$
D^{\mu}_{\nu} = \mathrm{diag}(\underbrace{+,\cdots,+}_{p+1},\underbrace{-,\cdots,-}_{9-p})\ . \qquad (3.6)
$$

The matrix D is included to account for the Dirichlet boundary condition on the world-sheet field associated with the directions perpendicular to the brane. Fixing the $SL(2,\mathbb{R})$ invariance of the amplitude (3.1), and performing the remaining integral, one finds that

$$
A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)}(sa_1 - ta_2), \qquad (3.7)
$$

where the two kinematic invariants are defined by

$$
s = 2p_{1\|}^2 = 2p_{2\|}^2,\ \ t = p_1\cdot p_2\ , \qquad (3.8)
$$

and

$$
\begin{aligned}
a_1 &= \mathrm{Tr}(\varepsilon_1 D)p_1\varepsilon_2 p_1 - p_1\varepsilon_2 D\varepsilon_1 p_2 - p_1\varepsilon_2\varepsilon_1^T Dp_1 - p_1\varepsilon_2^T\varepsilon_1 Dp_1 + \{1\leftrightarrow 2\}\\
&\quad -p_1\varepsilon_2\varepsilon_1^T p_2 - p_1\varepsilon_2^T\varepsilon_1 p_2 - s\mathrm{Tr}(\varepsilon_1\varepsilon_2^T)\ ,\\
a_2 &= \mathrm{Tr}(\varepsilon_1 D)(p_1\varepsilon_2 Dp_2 + p_2 D\varepsilon_2 p_1 + p_2 D\varepsilon_2 Dp_2) + p_1 D\varepsilon_1 D\varepsilon_2 Dp_2 + \{1\leftrightarrow 2\} \qquad (3.9)\\
&\quad +p_1 D\varepsilon_1\varepsilon_2^T Dp_2 + p_1 D\varepsilon_1^T\varepsilon_2 Dp_2 + s\mathrm{Tr}(\varepsilon_1 D\varepsilon_2 D) - s\mathrm{Tr}(\varepsilon_1\varepsilon_2^T)\\
&\quad -(s+t)\mathrm{Tr}(\varepsilon_1 D)\mathrm{Tr}(\varepsilon_2 D)\ .
\end{aligned}
$$

This string amplitude[5] is consistent with the well-known Dp-brane supergravity background geometry as shown in [11]. We note that the external closed string momenta are conserved only along the directions parallel to the branes, $p_{1\|} + p_{2\|} = 0$.

3.2 Turning on the B field

As we observed in section 2, the effect of the constant B field background can be incorporated simply by rotating the world-sheet fields by the matrix R defined there. Equivalently, we can

[5]We note that in Ref. [11], there is a typographical error that appears in the expression for a_1. For the scattering processes involving B, the correct formula shown here is important for the comparison with supergravity.

rotate the polarization tensors and momenta in the definitions of the vertex operators

$$V_1(z_1,\bar{z}_1) \;=\; (R^T\varepsilon_1 DR^T)_{\mu\nu}V^\mu_{-1}(R^Tp_1,z_1)V^\nu_{-1}(RDp_1,\bar{z}_1)\;, \tag{3.10}$$

$$V_2(z_2,\bar{z}_2) \;=\; (R^T\varepsilon_2 DR^T)_{\mu\nu}V^\mu_{0}(R^Tp_2,z_2)V^\nu_{0}(RDp_2,\bar{z}_2)\;, \tag{3.11}$$

while leaving the definitions (3.4), (3.5) as they are. A bit of algebra shows that the amplitude is modified as

$$A = \frac{\Gamma(\tilde{s})\Gamma(t)}{\Gamma(1+\tilde{s}+t)}(\tilde{s}a_1 - ta_2), \tag{3.12}$$

where $\tilde{s} = 2G^{\mu\nu}(p_\parallel)_\mu(p_\parallel)_\nu$, while the definition of t is the same as the one in Section 3.1. The polarization dependent part a_1 and a_2 are computed to be

$$\begin{aligned}
a_1 \;=\;& \mathrm{Tr}(\varepsilon_1 D_+)p_1\varepsilon_2 p_1 - p_1\varepsilon_2 D_+\varepsilon_1 p_2 - p_1\varepsilon_2\varepsilon_1^T D_- p_1 - p_1\varepsilon_2^T\varepsilon_1 D_+ p_1 + \{1\leftrightarrow 2\}\\
&-p_1\varepsilon_2\varepsilon_1^T p_2 - p_1\varepsilon_2^T\varepsilon_1 p_2 - \tilde{s}\mathrm{Tr}(\varepsilon_1\varepsilon_2^T)\;,\\
a_2 \;=\;& \mathrm{Tr}(\varepsilon_1 D_+)(p_1\varepsilon_2 D_+ p_2 + p_2 D_+\varepsilon_2 p_1 + p_2 D_+\varepsilon_2 D_+ p_2)\\
&+p_1 D_+\varepsilon_1 D_+\varepsilon_2 D_+ p_2 + \{1\leftrightarrow 2\}\\
&+p_1 D_+\varepsilon_1\varepsilon_2^T D_+ p_2 + p_1 D_+\varepsilon_1^T\varepsilon_2 D_+ p_2 + \tilde{s}\mathrm{Tr}(\varepsilon_1 D_+\varepsilon_2 D_+) - \tilde{s}\mathrm{Tr}(\varepsilon_1\varepsilon_2^T)\\
&-(\tilde{s}+t)\mathrm{Tr}(\varepsilon_1 D_+)\mathrm{Tr}(\varepsilon_2 D_+)\;.
\end{aligned} \tag{3.13}$$

To keep the notations simple, we introduced

$$D_\pm^{\mu\nu} \equiv D^{\mu\nu} + 2\Delta^{\mu\nu} \pm 2\Theta^{\mu\nu}\;, \tag{3.14}$$

and

$$\Delta^{\mu\nu} = G^{\mu\nu} - \eta^{\mu\nu}\;. \tag{3.15}$$

We note that $\tilde{s}$ is the s-channel momentum transfer computed with respect to the open string metric $G^{\mu\nu}$. When we push one of the bulk closed string vertex toward the open string boundary, we approach the s-channel factorization limit where we expect to observe intermediate open string states. As was explained in Ref. [7], these open string intermediate states feel the open string metric $G^{\mu\nu}$ instead of $\eta^{\mu\nu}$, providing an explanation for the new definition of $\tilde{s}$. In the large B limit, therefore, the kinetic energy along the directions parallel to the two-cycle along which B is turned on becomes negligible, being suppressed by $1/(1+B^2)$ factor (see also [17]).

3.3 Massless t-channel poles

The modifications due to the non-vanishing B field in Eqs. (3.13) when compared to Eqs. (3.9) become more transparent when we consider the leading t-channel poles. Essentially, the behavior of the string amplitudes around massless t-channel poles contains informations on the long range background fields. When expanded around massless t-channel poles, the scattering amplitude (3.12) reduces to

$$A \sim \frac{1}{t} a_1 + \mathcal{O}(1) \equiv \frac{1}{t}\bar{A} + \mathcal{O}(1) \ . \tag{3.16}$$

The bulk massless string states are gravitons, B fields and the dilaton. Therefore, for the two-point scatterings, there are six possible combinations of external closed string states. Among these, we write down below five possible combinations by explicitly plugging in the polarization states into Eqs. (3.13):

$$\begin{aligned} \bar{A}(B,\phi) &= -8p_1\Theta\varepsilon_1 p_2 \ , \\ \bar{A}(B,h) &= 2\{\mathrm{Tr}(\varepsilon_1\Theta)p_2\varepsilon_1 p_2 - 2p_2\varepsilon_1\Theta\varepsilon_2 p_1 + 2p_2\varepsilon_1\varepsilon_2\Theta p_2 - 2p_1\varepsilon_2\varepsilon_1\Theta p_1\}, \\ \bar{A}(\phi,\phi) &= 8\tilde{s}, \\ \bar{A}(\phi,h) &= 2(p-3)p_1\varepsilon_2 p_1 + 2\mathrm{Tr}(\Delta)p_1\varepsilon_2 p_1, \\ \bar{A}(B,B) &= -2p_1\varepsilon_2(D+2\Delta)\varepsilon_1 p_2 + 2p_1\varepsilon_2\varepsilon_1(D+2\Delta)p_1 \\ &\quad +2p_2\varepsilon_1\varepsilon_2(D+2\Delta)p_2 + 2p_1\varepsilon_2\varepsilon_1 p_2 - \tilde{s}\mathrm{Tr}(\varepsilon_1\varepsilon_2) \ . \end{aligned} \tag{3.17}$$

The arguments of $\bar{A}(x,y)$ denote the two external states x and y. The notable amplitudes are $\bar{A}(B,\phi)$ and $\bar{A}(B,h)$, which vanish when the background B field is set to zero [11]. ¿From the supergravity side, massless t-pole string amplitudes should be recovered by considering the three-point interactions through which the long range background field affects the external states. An inspection of the low energy supergravity action shows that the possible three-point interactions involving B fields are of the type B-B-graviton or B-B-dilaton. Therefore, the non-vanishing amplitudes $\bar{A}(B,\phi)$ and $\bar{A}(B,h)$ imply that there exists non-trivial long range background NS-NS two-form gauge field. When a constant B field is turned on on the brane world-volume, one might try to gauge it away to zero. However, for the directions parallel to the Dp-branes, we can not gauge it away, since it simultaneously involves the transformation of the world-volume $U(1)$ gauge field. The seemingly trivial constant B on the world-volume induces the non-trivial long range background B fields.

4 Comparison with Supergravity

In this section, we do the tree-level supergravity calculation of the two-point scatterings in the background geometries proposed in Refs. [9, 10]. The main result is that the leading t-pole string amplitudes computed in Section 3 are identical to the long-range supergravity tree amplitudes.

4.1 Supergravity background with or without B field

The NS-NS sector of the low energy effective action for Type II strings in ten dimensions reads, in Einstein frame,

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left\{R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}e^{-\phi}H^2\right\} . \tag{4.1}$$

The solutions representing a stack of parallel N Dp-branes are well-known and take the simplest form in the string frame:

$$ds^2 = H^{-1/2}(-dt^2 + \cdots + dx_p^2) + H^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2) , \tag{4.2}$$

$$e^{\phi} = H^{(3-p)/4} , \ \ H \equiv 1 + F \equiv 1 + (R_p/r)^{7-p} . \tag{4.3}$$

Recall that the Einstein metric and the string metric is related by $ds_E^2 = e^{-\phi/2}ds^2$. Recently, the authors of Refs. [9, 10] showed how to incorporate the effect of the B field background:

$$ds^2 = H^{-1/2}\left\{-dt^2 + \cdots + dx_{p-2k}^2 + \sum_{i=1}^{k} N_i(dy_{2i-1}^2 + dy_{2i}^2)\right\} + H^{1/2}dx_\perp^2 , \tag{4.4}$$

$$B_i = H^{-1}N_i \tan\theta_i , \tag{4.5}$$

$$e^{\phi} = H^{(3-p)/4}\prod_{i=1}^{k} N_i^{1/2} , \ \ N_i^{-1} \equiv \cos^2\theta_i + H^{-1}\sin^2\theta_i . \tag{4.6}$$

Their derivation is based on the chain of T-duality arguments suggested by [1]. Actually, the solutions of [9] are related to the solutions of [10] by a B-dependent rescaling of the y-coordinates. In our string calculations in Section 2, we chose $g_{\mu\nu} = \eta_{\mu\nu}$ and $\tan\theta_i = B_i$. We note that the solutions (4.6) are written in such a coordinate system that $e^{\phi} \to 1$, $ds^2 \to \eta_{\mu\nu}dx^\mu dx^\nu$ and $B_i \to \tan\theta_i$ near the asymptotic spatial infinity. Consistent with Section 2, there exist non-vanishing R-R background fields for the Dp-branes (proportional to $\cos\theta_i$) and for the D$(p-2)$-branes (proportional to $\sin\theta_i$) [10], which we do not consider in this paper[6].

4.2 Tree-level supergravity scatterings

We perform the analysis for the long-distance tree-level supergravity scatterings. Adding the background noncommutative D-branes to our problem is tantamount to adding a source term of the type [11]

$$S_{source} = \int d^{10}x\sqrt{-g}\{S_h^{\mu\nu}h_{\mu\nu} + S_\phi\phi + S_B^{\mu\nu}B_{\mu\nu}\} , \tag{4.7}$$

[6] To check the consistency of the background R-R gauge fields, we need to consider the two-point scatterings between NS-NS fields and R-R fields.

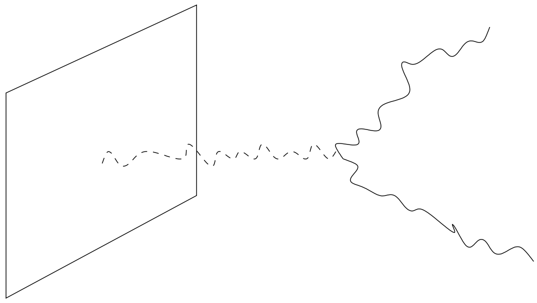

Figure 2: Typical supergravity t-channel scatterings. An incoming particle is scattered by the background fields to an outgoing particle.

where h, ϕ and B represent the fluctuations of graviton (around the flat background), dilaton and the NS-NS two-form gauge field, respectively. For source-probe type scatterings shown in Fig. 2, there are six possible external state combinations involving massless NS-NS particles. One can read off the three-point interaction vertices by expanding the low energy effective action (4.1) in flat spacetime background, and we find that there are four types of such interactions:

$$\begin{aligned} 4V(\phi,\phi,h) &= 2p_1\varepsilon_3 p_2 - p_1p_2\mathrm{Tr}(\varepsilon_3)\ , \\ 4V(B,B,\phi) &= 2p_1\varepsilon_2\varepsilon_1p_2 - p_1p_2\mathrm{Tr}(\varepsilon_1\varepsilon_2)\ , \\ 4V(B,B,h) &= -p_1\varepsilon_3p_2\mathrm{Tr}(\varepsilon_1\varepsilon_2) - 2p_1p_2\mathrm{Tr}(\varepsilon_1\varepsilon_3\varepsilon_2) \\ &\quad +2p_1\varepsilon_3\varepsilon_2\varepsilon_1p_2 + 2p_2\varepsilon_3\varepsilon_1\varepsilon_2p_1 + 2p_2\varepsilon_1\varepsilon_3\varepsilon_2p_1 \\ &\quad +\mathrm{Tr}(\varepsilon_3)\left[-p_1\varepsilon_2\varepsilon_1p_2 + \tfrac{1}{2}p_1p_2\mathrm{Tr}(\varepsilon_1\varepsilon_2)\right]\ , \end{aligned} \tag{4.8}$$

and $V(h,h,h)$. For the scatterings shown in Fig. 2, the leading non-trivial (r-dependent) background fields are nothing but

$$\text{background field of order } \frac{1}{r^{7-p}} = \text{source} \times \text{propagator}\ .$$

Therefore, we can replace the t-channel exchanged particle part of the diagram in Fig. 2 with its classical background field. By expanding the supergravity solutions of Refs. [9] and [10], i.e., Eqs. (4.4)-(4.6), the leading order background fields in Einstein frame are computed to be:

$$\begin{aligned} ds_E^2 &= ds^2_{\text{flat}}\{1+\tfrac{1}{8}F\mathrm{Tr}(\Delta)\} - F\Delta^{\mu\nu}dx^\mu dx^\nu + \mathcal{O}(F^2), \\ B_i &= \tan\theta_i + F\Theta_i + \mathcal{O}(F^2), \\ \phi &= -\tfrac{1}{4}F(p-3+\mathrm{Tr}(\Delta)) + \mathcal{O}(F^2)\ , \end{aligned} \tag{4.9}$$

where Θ_i and Δ are defined in Sec. 2 and Sec. 3.2, respectively. We compute (B,ϕ), (B,h), (ϕ,ϕ), (ϕ,h) and (B,B) scatterings using the vertex (4.8) and the background fields (4.9)[7].

[7] Though we have not computed the graviton-graviton scatterings, it is known from the analysis of [11] that (h,ϕ) and (B,B) scatterings are enough to uniquely determine the graviton and dilaton source terms in (4.7).

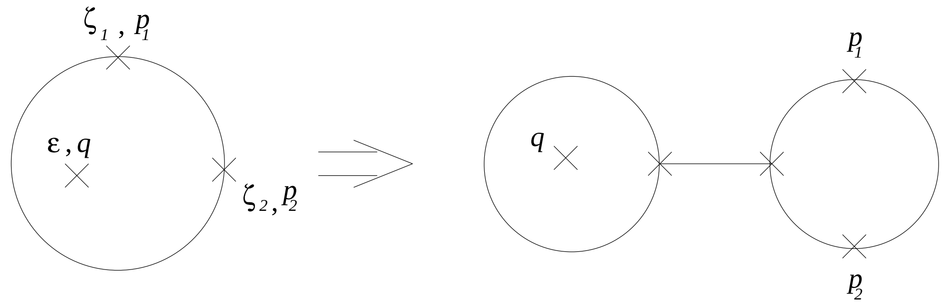

Figure 3: Three-point closed string absorption/emission diagram. On the right hand side, we show a factorization limit.

For each scattering, the relevant interaction vertices are $V(B,B,\phi)$, $V(B,B,h)$, $V(\phi,\phi,h)$, $V(\phi,\phi,h)$, $V(B,B,\phi)+V(B,B,h)$, respectively. The final results are identical to (3.16) and (3.17). In this fashion, our string theory calculations justify the supergravity solutions of Refs. [9, 10].

5 Absorption and emission by noncommutative D-branes

While the two-point scatterings computed in Section 3 can be used to justify the background geometries, the two-point scatterings do not exhibit the exponential phase factor used to define the $*$-product [7, 16]. The simplest non-trivial example showing such phase factor is the string emission/absorption diagram from/to noncommutative Dp-branes as shown in Fig. 3. In itself, this amplitude is important, for it shows nontrivial modifications of the Hawking radiation spectrum from the near-extremal noncommutative Dp-brane black holes, when compared to emissions from commutative Dp-brane black holes. Most notably, we find that the pole structure of the amplitude changes as we turn on the B field, and the exponential phase factors show up.

5.1 Review: Absorption/emissions from commutative Dp-branes

The amplitude is given by

$$A=\int_\Sigma d^2z\int_{\partial\Sigma}dw_1\int_{\partial\Sigma}dw_2\langle V_c(z,\bar z;q,\varepsilon)V_o(w_1;p_1,\zeta_1)V_o(w_2;p_2,\zeta_2)\rangle, \tag{5.1}$$

where the closed and open string vertex operators are

$$V_c(z,\bar z;q,\varepsilon) = (\varepsilon D)_{\mu\nu}V^\mu_{-1}(q,z)V^\nu_{-1}(Dq,\bar z)\ , \tag{5.2}$$

$$V_o(w;p,\zeta) = \zeta_\mu\{\partial X^\mu(w)+2ip\cdot\psi(w)\psi^\mu(w)\}e^{2ipX(w)}, \tag{5.3}$$

respectively. The kinematics of this scattering allows only one kinematic invariant t defined as $t = -2p_1 \cdot p_2$. The open string momenta p_1 and p_2 are restricted to lie along the D-brane world-volume, and the momentum is conserved along the directions parallel to the branes

$$p_1 + p_2 + q_{\|} = 0 \ . \tag{5.4}$$

Again, fixing the $SL(2, \mathbb{R})$ invariance and performing the integral, one finds

$$A = \frac{\Gamma(1-2t)}{\Gamma(1-t)^2} K, \tag{5.5}$$

where the kinematic factor is given by

$$\begin{aligned} K &= \mathrm{Tr}(\varepsilon D)(\zeta_1 q)(\zeta_2 D q) + 4(p_1 \varepsilon p_2)(\zeta_1 \zeta_2) + 2(\zeta_2 p_1)\{\zeta_1 \varepsilon D q + q D \varepsilon D \zeta_1\} \\ &\quad -4(\zeta_2 q)(\zeta_1 \varepsilon p_1) - 4(\zeta_2 D q)(p_1 \varepsilon D \zeta_2) + \{1 \leftrightarrow 2\} \\ &\quad +t\{\mathrm{Tr}(\varepsilon D)\zeta_1 \zeta_2 - 2\zeta_1 \varepsilon D \zeta_2 - 2\zeta_2 \varepsilon D \zeta_1\}. \end{aligned} \tag{5.6}$$

It was noted in Ref. [11] that this amplitude has poles for half integer values of t, but has *zeros* for integer values of t, summarized by a sort of "$\mathbb{Z}_2$ selection rule". In particular, there is no massless pole. As we will see shortly, the situation drastically changes as we turn on the B field.

5.2 Turning on the B field

We can include the effect of the background B field by placing the matrix R at appropriate places, as was done in Section 2. The closed string vertex operators change in the same way as in Eqs. (3.10) and (3.11) and the open string vertex operators become

$$V_o(w; Mp, M\zeta), \quad M \equiv \frac{1}{2}(R + R^T) \ . \tag{5.7}$$

Under these modifications, again paying attention to the fixing of the $SL(2, \mathbb{R})$ invariance, we obtain the following amplitude:

$$A = \frac{\Gamma(1-2t)}{\Gamma(1-t-\delta)\Gamma(1-t+\delta)} \left(\tilde{K} - \frac{\delta^2}{t} a \right) \ , \tag{5.8}$$

where we introduce

$$a = \mathrm{Tr}(\varepsilon D_+)\zeta_1 \zeta_2 \ , \quad \delta = 2 p_1 \Theta p_2 \ . \tag{5.9}$$

The polarization dependent quantity $\tilde{K}$ is obtained from K by the following rules: First, $\mathrm{Tr}(\varepsilon D)$ is replaced by $\mathrm{Tr}(\varepsilon D_+)$. Second, contractions of any two of the open string quantities

$(p_1, p_2, \zeta_1, \zeta_2)$ are made with respect to the open string metric $G^{\mu\nu}$. Accordingly, for example, t is now defined as $t = -2p_{1\mu}G^{\mu\nu}p_{2\nu}$. Third, contractions of two closed string quantities (ε, q) do not change. Finally, for contractions of an open string quantity and a closed string quantity, we insert $(G + \Theta)$. For example,

$$\zeta q \to \zeta_\mu (G + \Theta)^{\mu\nu} q_\nu. \tag{5.10}$$

The second and third rules are natural, since $G^{\mu\nu}$ is felt by open strings and the $\eta^{\mu\nu}$ is felt by closed strings.

An important feature of the amplitude (5.8) is that its pole structures are distinctively different from the commutative case, Eq. (5.5). We first consider a non-zero value of t. When δ is neither integral nor half-integral, the amplitude Eq. (5.8) has poles for both integral and half-integral values of t. When $t = m \pm \delta$ where $m = 1, 2, \cdots$, it has zeros. When δ is integral, the usual commutative $\mathbb{Z}_2$ selection rule applies: the amplitude has zeros for integral values of t and poles for half-integral values of t. When δ is half-integral, the situation reverses itself: the amplitude has zeros for half-integral values of t and poles for integral values of t. Near $t = 0$, we can use the formula

$$\Gamma(1 + x)\Gamma(1 - x) = \frac{\pi x}{\sin \pi x} \tag{5.11}$$

to find

$$A \sim \frac{1}{t}\bar{A}, \quad \bar{A} \sim \delta \sin(\pi\delta) \sim \delta(e^{i\pi\delta} - e^{-i\pi\delta}). \tag{5.12}$$

Eqs. (5.11) and (5.12) have further implication. By repeated use of $\Gamma(1 + x) = x\Gamma(x)$ and Eq. (5.11), the three-point amplitude Eq. (5.8) can be written as

$$A = A_0 \exp(i\pi\delta) - A_0 \exp(-i\pi\delta) \ , \tag{5.13}$$

where A_0 is an odd function under $\delta \to -\delta$, for integral values of t including $t = 0$. We notice that the exponential factors are exactly what one expects from the noncommutative Yang-Mills $*$-product. In Fig. 3, there are two possible factorization channels, one with p_1 on top (the shown figure) and another with p_2 on top (the flipped figure which is not shown). The amplitude (5.13), which is an even function under $\delta \to -\delta$ can be thought of as the sum of contributions from these two diagrams. The exponential factors, following [7] for the three open string insertions, will be

$$p_1\Theta p_2 + p_1\Theta q_\| + p_2\Theta q_\| = p_1\Theta p_2 - q_\|\Theta q_\| = p_1\Theta p_2 \ , \tag{5.14}$$

where we used the momentum conservation (5.4) along the parallel directions to the branes. Again using the same momentum conservation, we observe that δ can be considered as being linearly proportional to the external momentum $q_{||}$ and does not vanish as long as $q_{||}$ is not zero.

The appearance of the $*$-product phase factor for the external closed strings might seem peculiar: for the usual non-commutative Yang-Mills theory (i.e., essentially the pure open strings), such phase factors come from the planar diagrams where the external legs are attatched to the fundamental particles. Meanwhile, closed strings are described by composite operators[8]. One way to understand this peculiarity is to note that the phase factors in Eq. (5.13) show up in the factorization limit of the type shown in Fig. 3. The factorization limit of Fig. 3 is similar to the s-channel factorization limit of two-point scatterings shown in Fig. 1, where the closed string vertex operator approaches the open string boundaries. In this limit, the string absorption diagram contains a factor that corresponds to the three-point (open string) insertions along the open string boundaries, as shown in Fig. 3. This factor is responsible for the $*$-product phase factor. A related issue is to compute the string absorption amplitudes via classical supergravity analysis along the line of, for example, [20]. In the commutative case [20], it is known that string calculations are reproduced by the classical supergravity analysis. It is, however, unclear whether such classical supergravity analysis can reproduce the $*$-product phase factors shown in Eq. (5.13). Just like the s-channel limit of two-point scatterings, the $*$-product phase factor gets produced when the closed string vertex operator gets pushed very close to the open string boundaries. Unlike the t-channel limit of two-point scatterings, this is the limit where one has reasonable doubts about the validity of the perturbative classical supergravity. Even if it is a very interesting issue to see whether we can capture the $*$-product structure from the supergravity analysis, it is thus beyond the scope of this paper. We note that the exponential phase δ of Eq. (5.13) is taken to be vanishingly small in the supergravity limit [20].

6 Discussions

It is remarkable that the simple imposition of the boundary conditions as in Section 2 allows us to study rather intricate Dp-D$(p-2)$ bound states. Furthermore, the modifications that occur due to the constant $B_{\mu\nu}$ in the string calculations are straightforwardly accomplished by placing the matrix R (or R^T) of Section 2 at appropriate places of computations. There is an immediate technical generalization to the analysis presented in Section 2: one would like to extend the disk

[8]We thank the referee for pointing out this issue.

diagram analysis to the annulus diagrams, incorporating the open string loop effects.

The closed string absorption/emission amplitudes computed in Section 5 show some novel features, such as the occurrence of the exponential phase factors depending linearly on the external closed string momentum. Their pole and zero structures are also drastically modified as we turn on the B field. An interesting issue is to capture some of these features through the (possibly non-perturbative) supergravity absorption calculations along the line of, for example, Ref. [20]. This kind of comparison in fact gave some of motivations for the AdS/CFT correspondence conjecture for commutative D-branes [8]. In the context of noncommutative D-branes, the situation is much more subtle, for example, because the Dp-branes seemingly turn into D$(p-2)$-branes in the large B limit.

It is amusing to compare our calculation of the correlation functions to the gravitational back-reactions considered in Ref. [21]. The three-point amplitudes computed in Section 5 represent the absorption of NS-NS matter fields into noncommutative Dp-branes. As analyzed in [21], when almost light-like matter falls into a black hole, due to the energy conservation, the horizon radius gets increased; this effect translates to the shift of the horizon along the incoming null-direction and can thus be represented by the exponential phase proportional to the incoming momentum, or the shift operator. The three-point amplitudes show the similar exponential phase factor linearly proportional to the external momentum of the absorbed matter. The formal similarity might go further; the four-point amplitudes of type where there are two closed string vertex insertions in the bulk and two boundary open string vertex insertions has a factorization channel where two closed string vertices approach the open string boundary. In this limit, at least, it apparently appears possible to expect an exponential phase factor that depends quadratically on the external momenta, which, if exists, would correspond to the space-time noncommutativity induced by the world-volume noncommutativity. This type of four-point amplitudes represents the interaction between the emitted Hawking radiation and the absorbed matters. In Ref. [21], there are similar interaction effects summarized by the following exchange algebra, which corresponds to a version of space-time noncommutativity:

$$\phi_{in}(p_{in})\phi_{out}(p_{out}) = e^{i\kappa p_{in}\cdot p_{out}}\phi_{out}(p_{out})\phi_{in}(p_{in}), \tag{6.1}$$

where ϕ_{in} represents the absorbed matter and ϕ_{out} represents the emitted Hawking radiation. In Eq. (6.1), we notice an exponential phase factor that depends quadratically on the external momenta. It remains to be seen whether there is any reason why the space-time noncommutativity in Eq. (6.1) (due to the graviton exchanges near the black hole horizon) is formally similar to the possible space-time noncommutativity induced by the background B field.

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